Photorecombination of Electrons in Dense Plasmas

Young-Dae Jung and Soo-Yong Rhee
Department of Physics, Hanyang University, Ansan, Kyunggi-Do 425-791, South Korea
Reprint requests to Prof. Y.-D. J.; E-mail: yjung@bohr.hanyang.ac.kr

Z. Naturforsch. **55a**, 457–459 (2000); received November 5, 1999

The plasma screening effect on the photorecombination of free electrons, with ions in a weakly plasma is investigated. The recombination cross section is obtained by the principle of detailed balance with the photoionization cross section of the hydrogenic ion including the plasma screening effects on the bound and continuum states of the electron. It is found that the plasma screening effects on the recombination cross section are less than 11% when the Debye length (Λ) is greater than ten times of the Bohr radius (a_z) of the hydrogenic ion with nuclear charge Z.

Key words: Electron-ion Recombination; Plasma.

Photorecombination [1–6] is of interest in several areas of physics, such as astrophysics, atom and plasma physics, because it is the inverse process of photoionization. Photorecombination has also important consequences for X-ray astronomy since electron capture by ions is one of the important continuum X-ray emission mechanisms. The photorecombination cross sections of ions in dense plasmas may be different from those of free ions due to the screening effect of the surrounding plasma on the initial and final states of the electron. In dense laboratory and weakly coupled astrophysical plasmas the range of the Debey length Λ is known to be $\geq 10\, a_Z$, where $a_Z (\equiv a_0/Z = \hbar^2/Zme^2)$ is the first Bohr radius of a hydrogenic ion with nuclear charge Z, since the electron densities n_e and temperatures T_e are known to be around $10^{20} - 10^{23}$ cm^{-3} and $10^7 - 10^8$ K [7]. These plasmas can be classified as weakly coupled plasmas since the plasma coupling parameter (Γ) is quite a bit smaller than unity. The static Debye-Hückel model [7, 8] is known to be quite reliable to describe particle interactions in these dense weakly coupled plasmas. In this paper we investigate the plasma screening effect on photorecombination processes by ions in a dense weakly coupled plasma.

The inverse process to photorecombination is photoionization [9, 10], consisting in photon absorption and emission of an electron. Since we are dealing with binary collisions

$$A^{+Z+1} + e^- \leftrightarrow A^{+Z} + \hbar\omega, \tag{1}$$

the cross sections (σ_r: photorecombination, σ_i: photoionization) of both processes have the same dimension and are mutually related by the principle of detailed balance,

$$\frac{\sigma_r}{\sigma_i} = \frac{j_{\hbar\omega}}{j_{e^-}} \frac{g_r}{g_i}. \tag{2}$$

Here $j_{\hbar\omega} (= c/L^3)$ and $j_{e^-} (= v/L^3)$ are the fluxes of photons and electrons for the corresponding channels of the processes, normalized to one photon per a given cubic volume (L^3), where c is the speed of the light and v is the speed of the electron. In (2), g_r and g_i are the statistical weights of the final states of photorecombination and photoionization, respectively:

$$g_r = 2\, g_{A^{+Z}} \frac{4\pi k^2 dk}{(2\pi)^3}, \tag{3}$$

$$g_i = 2\, g_{A^{+Z+1}}\, g_{e^-} \frac{4\pi q^2 dq}{(2\pi)^3}, \tag{4}$$

where $g_{A^{+Z}}$, $g_{A^{+Z+1}}$, and g_{e^-} are statistical weights of the internal states of the particles, and k and q are the wave numbers of the photon and the electron, respectively. After some algebra, the cross section for the recombination of the free electron to the bound state of the ion with charge Z is found to be

$$\sigma_r = 2\frac{k^2}{q^2}\sigma_i. \tag{5}$$

Due to the energy conservation, $(k/q)^2$ can be represented by

$$\left(\frac{k}{q}\right)^2 = \frac{Z^2\alpha^2}{4} \frac{(\bar{E} + \bar{E}_b)^2}{\bar{E}}, \tag{6}$$

where α ($= e^2/\hbar c \approx 1/137$) is the fine structure constant, $\bar{E}$ ($\equiv mv^2/2Z^2$ Ry) the scaled electron energy, $\bar{E}_b$ ($\equiv E_b/Z^2$ Ry) the scaled binding energy, Ry ($= me^4/2\hbar^2 \approx 13.6$ eV) the Rydberg constant, and m the electron rest mass. For the recombination of the free electron to the 1s bound state of the ion with charge Z in weakly coupled plasmas, the binding energy E_b can be given by a recent investigation for the ground state energy and wave function including the plasma screening effect [7]:

$$E_b = Z^2 \text{ Ry } (1 - \delta_{1s}), \tag{7}$$

where δ_{1s} ($\cong 2a_\Lambda - 3a_\Lambda^2/2 + a_\Lambda^3$) represents the plasma screening correction to the 1s bound state energy and a_Λ ($\equiv a_Z/\Lambda$) is the reciprocal the scaled Debye length.

From (5) and (7), the radiative recombination cross section to the 1s bound state is given by

$$\sigma_{1s}^r = \frac{Z^2\alpha^2}{2\bar{E}}\left[\bar{E} + (1 - \delta_{1s})\right]^2 \sigma_{1s}^i, \tag{8}$$

where σ_{1s}^i is the 1s photoionization cross section [11] in weakly coupled plasmas described by the Debye-Hückel model potential, $V(r) = -\frac{Ze^2}{r}e^{-r/\Lambda}$, including the screening effect on the bound state, retardation correction, and screening effect on the continuum state:

$$\begin{aligned}\sigma_{1s}^r = (\bar{\varepsilon}, Z, a_\Lambda) = {} & \frac{2^9\pi^2}{3}\eta_{1s}^3(1-\delta_{1s})^{1/2}\alpha\, a_Z^2\,\bar{\varepsilon}^{-4} \\ & \times\left\{1 - Z^2\alpha^2\,\bar{\varepsilon}^2/[\bar{\varepsilon} - (1-\delta_{1s})]\right\}^{-2} \\ & \times\frac{e^{-4[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}\cot^{-1}[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}}}{1 - e^{-2\pi[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}}} \\ & \times\sqrt{\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}-(1-\delta_{1s})}}\left(\tan^{-1}\sqrt{\bar{\varepsilon}-1}\right)^{-2} \\ & \times\left[\tan^{-1}\left(\frac{\sqrt{\bar{\varepsilon}-(1-\delta_{1s})}}{\eta_{1s}+a_\Lambda}\right)\right. \\ & \left. + \frac{a_\Lambda\sqrt{\bar{\varepsilon}-(1-\delta_{1s})}}{(\eta_{1s}+a_\Lambda)^2 + \bar{\varepsilon} - (1-\delta_{1s})}\right]^2. \end{aligned} \tag{9}$$

The parameter η_{1s} ($\cong 1 - 3a_\Lambda^2/4 + a_\Lambda^3$) represents the plasma-screening effect on the 1s Bohr radius and $\bar{\varepsilon}$ ($\equiv \hbar\omega/Z^2$ Ry) is the scaled photon energy. The term

$$\sqrt{\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}-(1-\delta_{1s})}} \times \frac{e^{-4[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}\cot^{-1}[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}}}{1 - e^{-2\pi[\bar{\varepsilon}-(1-\delta_{1s})]^{-1/2}}}$$

in (9) represents the plasma screening effect on the continuum state. In obtaining (9), we have restricted ourselves to hydrogenic wave functions with $Z\alpha \ll 1$, so that relativistic effects for bound state wave functions were neglected since the relativistic corrections are only of relative order $(Z\alpha)^2$. The 1s state wave function and energy eigenvalue of the hydrogenic ion were obtained using the Ritz variation method [6]. Using the energy conservation, $\bar{\varepsilon} - 1 = \bar{E} - \delta_{1s}$, the 1s photorecombination cross in units of πa_0^2 as a function of the electron energy ($\bar{E}$) is then found to be

$$\begin{aligned}\sigma_{1s}^r(\bar{E}, Z, a_\Lambda)/\pi a_0^2 & \\ = \frac{2^8\pi}{3}\alpha^3\eta_{1s}^3 & (1-\delta_{1s})^{1/2}\frac{\sqrt{\bar{E}-\delta_{1s}}}{\bar{E}^{3/2}\left[\bar{E}+(1-\delta_{1s})\right]^2} \\ & \times\left\{1 - \frac{Z^2\alpha^2\left[\bar{E}+(1-\delta_{1s})\right]^2}{\bar{E}}\right\}^{-2} \\ & \times\frac{e^{-4\bar{E}^{-1/2}\cot^{-1}\bar{E}^{-1/2}}}{1 - e^{-2\pi\bar{E}^{-1/2}}}\left(\tan^{-1}\sqrt{\bar{E}-\delta_{1s}}\right)^{-2} \\ & \times\left[\tan^{-1}\left(\frac{\bar{E}^{1/2}}{\eta_{1s}+a_\Lambda}\right) + \frac{a_\Lambda\bar{E}^{1/2}}{(\eta_{1s}+a_\Lambda)^2+\bar{E}}\right]^2. \end{aligned} \tag{10}$$

In (10), $\sqrt{(\bar{E}-\delta_{1s})/\bar{E}}\; e^{-4\bar{E}^{-1/2}\cot^{-1}\bar{E}^{-1/2}}/(1 - e^{-2\pi\bar{E}^{-1/2}})$ represents the plasma screening effect and Coulomb correction on continuum state as a function of the scaled projectile energy. Hence, the screening effects on the initial continuum and the final bound states are included in (10). Even though the screened Coulomb correction is obtained by the radial integration from zero to infinity rather than from zero to the Debye length Λ, the result (10) is quite reliable since the Debye length is greater than ten times of the Bohr radius when the electron densities n_e and temperatures T_e are around 10^{20}–10^{23} cm^{-3} and 10^7–10^8 K, respectivey, and the exponential factor $e^{-r/\Lambda}$ in the radial matrix element is quite small for $r \approx \Lambda$. Since the recombination cross section decreases rapidly with increasing principle quantum number n, the electron is most likely captured into the ground state [1]. The total recombination is known to be important at intermediate temperatures, particularly when the plasma consists mostly of hydrogenic ions.

In order to explicitly investigate the total plasma screening effect on the 1s photorecombination cross section, specifically, we consider three cases of the Debye length: $a_\Lambda = 0.1, 0.05$, and 0, i.e., $\Lambda = 10a_z, 20a_z$, and ∞, and we assume that $Z = 2$ since our nonrelativistic result (10) is valid for $Z\alpha \ll 1$. Figure 1 shows the photorecombination cross section for the electron capture into the 1s bound state as a function of the scaled electron energy $\bar{E}$ ($\equiv mv^2/2Z^2$ Ry). The unscreened photorecombina-

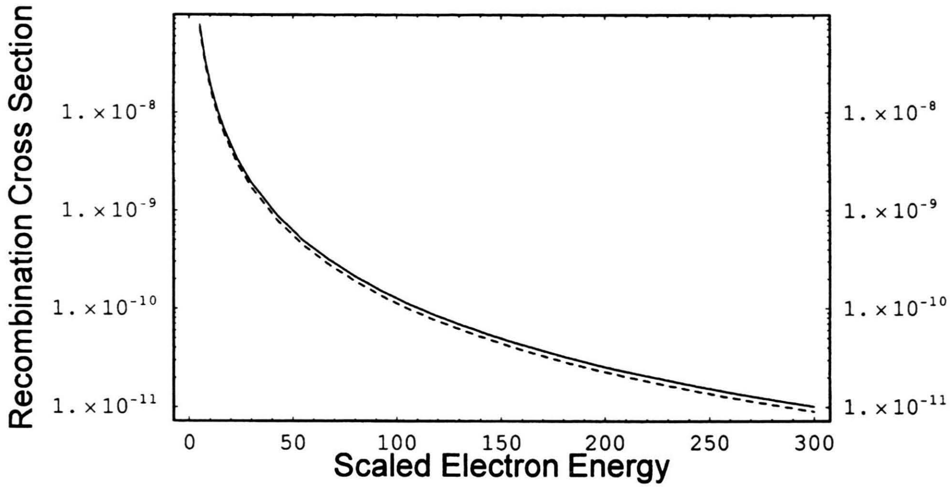

Fig. 1. The 1s photorecombination cross sections σ_{1s}^{r} in units of πa_0^2 for $Z = 2$ as functions of the scaled electron energy $\bar{E}$ (= $mv^2/2Z^2$ Ry). The dashed line represents the photorecombination cross section for $a_\Lambda = 0.1$, i.e., including the plasma screening effects. The solid line represents the photorecombination cross section for $a_\Lambda = 0$, i.e., neglecting the plasma screening effects.

Table 1. The numerical values of the 1s photorecombination cross sections in units of πa_0^2 for $Z = 2$.

a_Λ	σ_{1s}^{r} $(\bar{E} = 50)^{a}$ (πa_0^2)	σ_{1s}^{r} $(\bar{E} = 150)^{b}$ (πa_0^2)
0.1	5.52267×10^{-10}	4.42398×10^{-11}
0.05	5.88134×10^{-10}	4.72195×10^{-11}
0	6.19944×10^{-10}	4.98809×10^{-11}

[a] Photorecombination cross section for $\bar{E}$ (= $mv^2/2Z^2$ Ry) = 50.
[a] Photorecombination cross section for $\bar{E}$ (= $mv^2/2Z^2$ Ry) = 150.

tion cross section is also illustrated. As we can see in this figure, the plasma screening effects are almost independent of the incident electron energy for a given Debye length. The numerical values of the photorecombination cross sections in units of πa_0^2 are listed in Table 1. The plasma screening effects reduce the photorecombination cross sections (e.g., ≈ 11% for $a_\Lambda = 0.1$, ≈ 5% for $a_\Lambda = 0.05$).

In conclusion, we have investigated the plasma screening effects on photorecombination of the free electron to the 1s bound state of the hydrogenic ion in a dense weakly coupled plasma. The charged particle interaction potential in plasma is given by the static Debye-Hückel model. The plasma screening effect on the photorecombination cross section is obtained as a function of the Debye length and electron energy. It is found that the plasma screening effects on the photorecombination cross section for the interesting domain of the Debye length, $\Lambda \geq 10 a_z$, are less than 11%. These results provide a useful information for radiative recombination processes in dense plasmas.

Acknowledgements

One of the authors (Y.-D. Jung) is grateful to Prof. H. Tawara for helpful discussions and useful comments. The authors wish to acknowledge the financial support of Hanyang University, South Korea, made in the program year of 1999. This work was supported by the Korean Ministry of Education through the Brain Korea (BK21) Project and by the Korea Basic Science Institute through the HANBIT User Development Program (FY2000).

[1] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, Plenum, New York 1977, Chapt. 6.
[2] R. J. Gould and R. K. Thakur, Ann. Phys. **61,** 351 (1970).
[3] R. J. Gould, Astrophys. J. **219**, 250 (1978).
[4] W. H. Tucker, Radiation Processes in Astrophysics, MIT Press, Cambridge 1975, Chapt. 6.
[5] D. E. Osterbrock, Astrophysics of Gaseous Nebulae and Galactic Nuclei, University Science Book, Mill Valley 1989, Chapt. 2.
[6] V. P. Krainov, H. R. Reiss, and B. H. Smirnov, Radiative Processes in Atomic Physics, Wiley, New York 1997, p. 165.
[7] Y.-D. Jung, Phys. Fluids B **5,** 3432 (1993).
[8] Y.-D. Jung, Phys. Plasmas **2,** 332 (1995).
[9] W. K. Rose, Advanced Stellar Astrophysics, Cambridge University Press, Cambridge 1998, Chapt. 4.
[10] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Quantum Electrodynamics, Pergamon Press, Oxford 1982, Chapt. 5.
[11] Y.-D. Jung, Phys. Plasmas **5,** 3781 (1998).